

$$\textcircled{1} \quad \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 2 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 2 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 2 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \xrightarrow{L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -\frac{1}{2} & 1 \end{pmatrix}} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \xrightarrow{U = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\textcircled{2} \quad \begin{array}{l} x + 2y - 2z = 5 \\ 2x + 4y - 4z = 5 \end{array} \xrightarrow{\text{normale.}} \begin{array}{l} A^T A x = A^T b \\ \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} 1 & 2 & -2 \\ 2 & 4 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix} \end{array} \xrightarrow{\text{rref}} \begin{pmatrix} 1 & 2 & -2 & | & 3 \end{pmatrix}$$

$$\text{det. mo: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2s+2t \\ s \\ t \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}s + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}t \quad \boxed{1} \quad \text{sortierbar zu spezif. } (c, 2c, -2c) \rightarrow c = \frac{1}{3} \begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix}$$

$$\boxed{2} \quad \begin{pmatrix} 1 & 2 & -2 & | & 3 \\ -2 & 1 & 0 & | & 0 \\ 2 & 0 & 1 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & \frac{1}{3} \\ 0 & 1 & 0 & | & -2/3 \\ 0 & 0 & 1 & | & -2/3 \end{pmatrix} \quad \text{entferne mo: } \begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}s + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}t \quad \boxed{1} \quad \boxed{2} \quad \boxed{5}$$

$$\textcircled{3} \quad A = \begin{pmatrix} 1 & 1 & 3 & 2 & 3 \\ 1 & 2 & 5 & 2 & 2 \\ 2 & 3 & 8 & 4 & 5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 3 & 2 & 3 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 & 2 & 4 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 \end{pmatrix} \quad \text{rang } A = 2 \quad \boxed{1} \quad A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 2 & 4 \\ 0 & 1 & 2 & 0 & 1 \end{pmatrix} \quad \boxed{1} \quad \boxed{4}$$

$$\textcircled{4} \quad \boxed{1} \quad \text{sorrendi kifejtés: } \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 0 \\ 2 & 0 & 1 \end{vmatrix} = 1 - 2 + 2 = 1 \quad \boxed{1} \quad = \begin{pmatrix} 1 \\ 2 \end{pmatrix} [1 \ 0 \ 1 \ 2 \ 4] + \begin{pmatrix} 1 \\ 2 \end{pmatrix} [0 \ 1 \ 2 \ 0 \ 1] \quad \boxed{1}$$

$$\boxed{2} \quad \text{elemi soránváltás: } \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & -1 & 1 \\ 0 & 0 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & -2 & 1 \end{vmatrix} = 1 \quad \boxed{2}$$

$$\boxed{3} \quad \text{egyséle összegese bontás: } \boxed{1} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \boxed{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 1 + 2 - 2 = 1 \quad \boxed{2} \quad \boxed{5}$$

$$\textcircled{5} \quad \begin{array}{c} \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{pmatrix} \\ \xrightarrow{\substack{(1) \rightarrow \\ (1) \leftrightarrow (2) \\ (1) \leftrightarrow (3)}} \begin{pmatrix} \cos \frac{2\pi}{3} & \cos \frac{\pi}{6} & 0 & 1 \\ \cos \frac{\pi}{6} & \cos \frac{\pi}{3} & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \approx \frac{2\pi}{3} & \approx \frac{\pi}{6} & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \frac{4\pi}{3} & \approx \frac{2\pi}{3} & 0 & 1 \\ \cos \frac{\pi}{6} & \cos \frac{2\pi}{3} & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \approx \frac{2\pi}{3} & \approx \frac{\pi}{6} & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 1 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 1 \end{pmatrix} \end{array} \quad \boxed{2} \quad \boxed{7} \quad \begin{pmatrix} 1 & 1 & 1 & | & 1 & 0 & 1 \\ 0 & 1 & 1 & | & 1 & 0 & 0 \\ 1 & 0 & 1 & | & 0 & 1 & 0 \\ 1 & 1 & 0 & | & 1 & 0 & 0 \end{pmatrix} \xrightarrow{\substack{(1) \leftrightarrow (2) \\ (1) \leftrightarrow (3) \\ (1) \leftrightarrow (4)}} \begin{pmatrix} 1 & 1 & 1 & | & 1 & 0 & 1 \\ 1 & 2 & -1 & | & 2 & 1 & 0 \\ 1 & 1 & 0 & | & 1 & 0 & 0 \\ 2 & 1 & -1 & | & 2 & 1 & 0 \end{pmatrix} \quad \boxed{2}$$

$$\textcircled{6} \quad \begin{array}{c} \text{null det trace} \\ \text{forg 2 vertikális} \\ \begin{pmatrix} 1 & & & \\ 3 & 0 & 1 & 2 \cos \alpha + 1 \\ 2 & 1 & 0 & 2 \end{pmatrix} \end{array} \quad \boxed{2} \quad \boxed{4} \quad = \frac{1}{3} \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} \quad \boxed{1} \quad \sqrt{1+1+1} = \sqrt{3} \quad \boxed{1} \quad \boxed{4}$$

$$\textcircled{8} \quad \boxed{1} \quad \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \quad \boxed{1} \quad \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \boxed{1} \quad \boxed{2} \quad \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix} \quad \boxed{4}$$

$$\textcircled{9} \quad \begin{pmatrix} 2-x & 0 & 0 \\ 1 & 1-x & -2 \\ 2 & -2 & -2-x \end{pmatrix} = (2-x) \underbrace{\begin{pmatrix} (1-x)(-2-x)-4 \\ x^2+x-6 \end{pmatrix}}_{= -x^3 + x^2 + 8x - 12} = (2-x)(2-x)(-3-x) \quad \boxed{2}$$

$$\textcircled{7} \quad \boxed{2} \quad X \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \rightsquigarrow X = \frac{1}{3} \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & 1 & 2 \end{pmatrix} \quad \boxed{2} \quad \boxed{4} \quad \begin{pmatrix} 0 & 0 & 0 \\ 1 & -1 & -2 \\ 2 & -2 & -4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \boxed{2}$$

$$axb = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \boxed{1} \quad \boxed{5}$$