1. Determine the matrix of the following linear transformations/maps in the given basis/pair of bases. Determine also the basis of the kernel and the image.

(i) $f : \mathbb{R}^3 \to \mathbb{R}^2$, $f((x,y,z)^T) = (x+2y-z, x-y+z)^T$ in the standard bases, and in the pair of bases $\mathcal{B} = \{(1,1,0)^T, (0,1,1)^T, (1,0,1)^T\}$; $\mathcal{C} = \{(1,1)^T, (2,3)^T\}$.

(ii) The rotation around the axis $x = t$, $y = 2t$, $z = -t$ ($t \in \mathbb{R}$) by $90°$ in the standard basis.

(iii) The transformation $A \mapsto A + A^T$ among 2×2 real matrices in the "standard" basis of matrices.

(iv) The multiplication by $z = 2e^{\frac{2\pi i}{3}}$ in the $\mathbb{C}_\mathbb{R}$ real vector space in the basis $\mathcal{B} = \{1, i\}$ and also in the basis $\mathcal{C} = \{1, e^{\frac{2\pi i}{3}}\}$.

(v) Given a (finite) set X, let $\mathbb{R}^X$ denote the space of real valued functions $X \to \mathbb{R}$. Let now the linear transformation $\varphi : \mathbb{R}^X \to \mathbb{R}^X$ be defined by

$$\varphi(f)(x) = \sum_{x \neq y \in X} f(y),$$

for an $f \in \mathbb{R}^X$. Determine the matrix of φ in the basis of $\mathbb{R}^X$ consisting of the characteristic functions of the points $\mathbb{1}_x$ ($x \in X$), that is

$$\mathbb{1}_x(y) = \delta_{yx} = \begin{cases} 1, & \text{if } y = x, \\ 0 & \text{otherwise.} \end{cases}$$

2. Which linear transformations are basis-insensitive? (That is, the matrix is the same, irrespective of the basis.)