Introduction to Algebra **Study room 7** **Spring 2025**

1. Which of the following are vector spaces (and what is the underlying field)? If not then specify at least one axiom which fails.

(i) $\{f \in \mathbb{R}[x] \mid \deg(f) = 100 \text{ or } f = 0\}$;

(ii) $\{f \in \mathbb{R}[x] \mid \deg(f) \leq 100 \text{ or } f = 0\}$;

(iii) $\{f \in \mathbb{R}[x] \mid \deg(f) \geq 100 \text{ or } f = 0\}$;

(iv) polynomials with 0 constant term;

(v) polynomials such that the sum of the coefficients is 0;

(vi) polynomials that give constant remainder divided by $x^2 + 1$;

(vii) polynomials such that $a_i = 0$ if i is odd;

(viii) real series with first term 0;

(ix) real series with all, but finitely many terms being 0;

(x) real series with infinitely many 0 terms;

(xi) rational series;

(xii) matrices with entries 0 below the main diagonal (upper triangular matrices);

(xiii) matrices with entries 0 below or on the main diagonal (strictly upper triangular matrices);

(xiv) upper bounded functions;

(xv) continuous functions;

(xvi) periodic functions;

(xvii) matrices A such that $A = -A^T$ (skew-symmetric matrices);

(xviii) subsets of a set S with addition $A + B = (A \setminus B) \cup (B \setminus A)$.

2. Which of the following subsets of $\mathbb{R}^3$ is a subspace?

a) $\{v \in \mathbb{R}^3 | \ |v| = 1\}$, b) $\{(x, y, z) | x + 2y + z = 0\}$ and c) $\{(x, y, z) | x + 2y + z = 1\}$.

NB. Not every line/plane is a subspace. There are *affine subspaces* of the form $u + W$, where u is a vector and W is a subspace.