Introduction to Algebra **Study room 6** **Spring 2025**

1. True or false?

(i) If $f(x) \in \mathbb{Z}[x]$ of degree 3 has 3 rational roots then product of the roots is an integer.

(ii) If $f(x) \in \mathbb{Z}[x]$ of degree 2 has 2 real roots then the difference of the roots is an integer.

(iii) If $f(x) \in \mathbb{C}[x]$ of degree n and every power sum of its n roots is an integer then $f(x) \in \mathbb{Q}[x]$.

2. Express the following polynomials using the elementary symmetric polynomials

(i) $x^5y^2 + x^2y^5$ in 2 variables,

(ii) $x^3y^2 + x^3z^2 + x^2y^3 + x^2z^3 + y^3z^2 + y^2z^3$ in 3 variables and

(iii) $t^8 + u^8$ in 2 variables.

3. Suppose a, b, c are thee real numbers such that $a+b+c = 2$, $a^2+b^2+c^2 = 6$ and $abc = -2$. Provide a (monic) polynomial whose roots are exactly a, b, c.

4. * Suppose the real polynomial $f(x) = x^3 + px + q$ has three real roots. What should hold for the coefficients p, q?