1. True or false?

(i) If $f(x) \in \mathbb{Z}[x]$ has an integer root then it is not irreducible in $\mathbb{Q}[x]$.

(ii) If $f(x) \in \mathbb{Z}[x]$ is not irreducible in $\mathbb{Q}[x]$ then it not irreducible in $\mathbb{Z}[x]$.

(iii) If $f(x) \in \mathbb{Z}[x]$ and it has a rational root then it has a degree 1 divisor in $\in \mathbb{Z}[x]$.

(iv) If $f(x) \in \mathbb{Q}[x]$ and $f(n)$ is an integer for every $n \in \mathbb{Z}$ then $f(x) \in \mathbb{Z}[x]$.

2. Find the roots of the following polynomials and decompose to product of irreducibles over $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{F}_5$!

(i) $2x^3 - 7x^2 + 2$,

(ii) $x^6 - 2x^5 - x^4 + 4x^3 - 5x^2 + 6x - 3$ and

(iii) $x^5 + 1$.

3. Decompose $x^p - 1 \in \mathbb{F}_p[x]$ into a product of irreducible polynomials.