1. True or false?

(i) If f(x) has degree n then it has at least n distinct roots.

(ii) If f(x) has n distinct roots then it has degree n.

(iii) If f(x) has degree n then it has at most n distinct roots.

(iv) $\prod_{a \in \mathbb{F}_p} (x - a) = x^p + \dots + (-1)x + 0 \in \mathbb{F}_p[x].$

2. Compute the value of $p(x) = x^5 - 3x^2 + x + 3$ at x = 5 using Horner's method! Divide p(x) by x - 5. What is the quotient and what is the remainder?

3.

- (i) What is $5^{-1} \pmod{26}$?
- (ii) Is 4 invertible modulo 26?

4. Compute the table of the operations of \mathbb{F}_3 and \mathbb{Z}_8 ! What is the table of multiplication in \mathbb{Z}_8^* ?

5. Divide $x^4 - 2x + 3$ with remainder by

- (i) $x^2 x + 2$,
- (ii) x + 1,
- (iii) $(x+1)^2$,
- (iv) $x^2 1$.

6. Show that every polynomial in $\mathbb{R}[x]$ of odd degree has at least one real root!