Introduction to Algebra **Study room 2** **Spring 2025**

1. True or false?

(i) If $a \equiv b \pmod{c}$, where $3 \mid b$ but $3 \nmid c$ then $3 \mid a$.

(ii) If $a \equiv b \pmod{c}$, where $3 \nmid b$ but $3 \mid c$ then $3 \mid a$.

(iii) If $a \equiv b \pmod{c}$, where $3 \mid b$ and $3 \mid c$ then $3 \mid a$.

2. What is the remainder of 99! modulo 10100?

3. Suppose $m > 3$ is composite. What is the remainder of $(m-1)!$ modulo m?

4. Two centipedes compete, which has more legs.

(i) The first counts them by 11-s and 5 remain uncounted. Then groups them by 15-s then 3 remain ungrouped. How many legs does it have?

(ii) The second counts them by 12-s and 4 remain uncounted. Then groups them by 15-s then 8 remain ungrouped. Surely something went wrong!

5. Suppose

$$\begin{aligned} x &\equiv 6 \pmod{7} \\ x &\equiv 3 \pmod{11} \\ x &\equiv 9 \pmod{13} \end{aligned}$$

What is $x \equiv? \pmod{1001}$?

6. What are the last three digits of 1234^{9876}?