1. Which of the following are bilinear functions on the real polynomials of degree at most 3? If bilinear then determine its matrix in the canonical $\{1, x, x^2, x^3\}$ basis.

(i) $B_1(f, g) = fg$;

(ii) $B_2(f, g) = f(1) + g(1)$;

(iii) $B_3(f, g) = f(1)g(2)$;

(iv) $B_4(f, g) = f'(1)g(1)$;

(v) $B_5(f, g) = $ the coefficient of x^2 in fg.

2. Diagonalise the bilinear functions given by the following matrices and determine their definiteness.

$$A_1 = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}; \quad A_2 = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}; \quad A_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{pmatrix}$$

3. Determine the possible Jordan Normal Forms of complex 2×2 matrices with

(i) unique eigenvalue λ;

(ii) two unequeal eigenvalues λ, μ.