1. Determine the Jordan Normal Form of the following matrices

$$E = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & -1 \end{bmatrix};\ F = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 0 \\ -4 & 0 & 2 \end{bmatrix};\ G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

2. Suppose $A \in M^{16\times 16}(\mathbb{C})$ and the rank-sequence of the powers of $A - I$, $A - 2A$ and $A + I$ are in the following table. Determine the Jordan Normal Form of A.

	1	2	3	4	5	6
$A - I$	14	13	12	11	10	9
$A - 2I$	14	12	11			
$A + I$	13	12				

2. Compute the powers of the following Jordan matrices (blocks)

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix};\ B = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix};\ C = \begin{bmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{bmatrix};\ D = \text{same for } n \times n.$$

4. What is the Jordan Normal form of a projection?

5. Let A be a square matrix and suppose $\chi_A(t) = t^n$ and $m_A(t) = t^k$. Let $r_i = \mathrm{rk}(A^i)$ denote the sequence of the ranks of the powers of A.

(i) Determine r_0 and r_k.

(ii) Verify that $r_{i+1} < r_i$ (for $0 \le i < n$), that is the sequence is strictly decreasing.

(iii) Prove that $r_i \le \frac{r_{i-1}+r_{i+1}}{2}$ (for $0 < i < n$), that is the sequence is weakly convex.

(iv) Show that if a sequence r_i satisfies the above three properties then there exists a matrix A for which $r_i = \mathrm{rk}(A^i)$ (for $0 \le i \le n$).

6. HW Compute the minimal polynomial and the JNF of the following matrix. Provide a Jordan basis, too.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

7. Suppose $A, B \in M^{n\times n}(\mathbb{R})$ and $C \in M^{n\times n}(\mathbb{C})$ such that $A \sim C$ and $B \sim C$ over $\mathbb{C}$. Show that $A \sim B$ over $\mathbb{R}$.
In particular, the real matrices A, B have the same Jordan Normal Form (over $\mathbb{C}$) if and only if they are similar. (We can also replace "real" by "rational.")

8. Find one representative from each $\sim$-equivalence class from $M^{2\times 2}(\mathbb{F}_2)$. Do it in $M^{3\times 3}(\mathbb{F}_2)$, too.