1. Equivalence relation. Examples. Equivalence Relation Theorem.

2. Integers. Division, division with remainder. Euclid's algorithm, gcd, lcm. Solvability of $(a, b) = ax + by$. Units, irreducible and prime integers, Fundamental Theorem of Arithmetic.

3. Numbers at different bases. Horner's method. Congruences, Diophantine equations. Chinese Remainder Theorem. Complete and reduced systems of residues. Euler's totient function. Wilson's Theorem. Euler-Fermat Theorem and Fermat's Little Theorem.

4. Polynomials over fields. Evaluation map, roots. Divisibility, long division, division with remainder. Root factor. Number theory of polynomials and the unique factoristion property.

5. Multiplicity of roots and testing with the derived polynomial. Degree n polynomial has at most n roots. Vieta's formuli. Symmetric polynomials, elementary symmetric polynomials. Fundamental Theorem of Symmetric Polynomials. Newton's formuli.

6. Polynomials over the integers. Rational root test, primitive polynomial. Gauss' Lemma (three with this name). Schönemann-Eisenstein criterion.

7. Mod p field $\mathbb{F}_p$ and $\mathbb{F}_p[x]$.

8. Complex polynomials. Fundamental theorem of Algebra (algebraically closed field). Roots of 1, order, primitive roots of 1. Cyclotomic polynomials. Irreducibility of $\Phi_p(x)$ over the rationals (special case of Gauss' Theorem).

9. Vector space over any field. Subspaces. Linear combination, generated subspace, closure conditions. Sum and direct sum. Dependence, independence. Four properties. Infinite sets.

10. Basis. Replacement theorem, existence of basis if the vector space is finitely generated. Dimension. Zorn's Lemma, basis and dimension of infinitely generated vector spaces.

11. Linear maps, transformations, $\mathrm{Hom}(V_1, V_2)$, $\mathrm{Hom}(V)$. Properties, kernel, image. Description of φ by images of basis vectors. Isomorphism, dimension.

12. Dimension theorem: If $\varphi : V_1 \to V_2$ then $\dim(\mathrm{Ker}(\varphi)) + \dim(\mathrm{Im}(\varphi)) = \dim(V_1)$. $\mathrm{Hom}(V_1, V_2)$ is a vector space, $\mathrm{Hom}(V)$ is also a ring (actually an F-algebra). Projections.

13. Matrix of a linear map/transformation. Multiplication of matrices and maps. Dimension of $\mathrm{Hom}(V_1, V_2)$, $\mathrm{Hom}(V)$. Change of basis, similarity of matrices in $M^{n\times n}(F)$. Invariance of trace, determinant, rank and nullity. $\mathrm{tr}(AB) = \mathrm{tr}(BA)$.

14. Eigenvector, eigenvalue, eigenspace of linear transformations. Eigenbasis, diagonalisability. $f(\varphi)$ and $f(A)$ for $f \in F[x]$, $\varphi \in \mathrm{Hom}(V)$ and $A \in M_n(F)$.

15. Minimal polynomial and its roots. Characteristic polynomial and its roots. Cayley-Hamilton Theorem. Algebraic and geometric multiplicities.

16. Block matrices, invariant subspaces, nilpotent matrices, generalised eigenspaces. and the Jordan Normal Form. Jordan chains for a given eigenvalue. JNF theorem in two forms.

17. Bilinear functions and its matrix in a basis. Change of basis. Perpendicularity($\perp$), left/right perpendicular subspaces, nondegenerate bilinear functions. Symmetric and symplectic functions.

18. Symmetric bilinear functions are diagonalisable (if $2 \neq 0$ in the field). Different diagonalisation algorithms. Sylvester's Law of Intertia over $\mathbb{R}$, signature. Quadratic form and its definiteness.

19. Sesquilinear functions over $\mathbb{C}$, matrix form, adjoint matrix. Hermitian functions. Diagonalising Hermitian functions, Sylvester's Law of Intertia. Complex quadratic form and its definiteness.

20. Real and complex Euclidean spaces. Scalar/inner product, length, angle (only for real). Bunyakovsky-Cauchy-Schwarz inequality and the triangle inequality. Metric space. Norm.

21. Adjoint of a transformation, $U \leq V$ is φ-invariant if and only if $U^{\perp}$ is φ^*-invariant. Symmetric/self-adjoint, orthogonal/unitary and normal transformations. Orthogonal eigenspaces, real eigenvalues. Complex Spectral Theorem: Complex ON eigenbasis $\Leftrightarrow$ normal. Equivalent descriptions of unitary transformations and similar equivalent descriptions of orthogonal transformations.

22. Complex versions of polar decomposition, SVD and the pseudoinverse.

23. Applications. Linear maps to find special polynomials. Hoffman-Singleton theorem.