

1. Groups, basic properties, examples, cyclic groups
2. Order of an element, set product, Cayley table
3. Subgroup, proper subgroup. Cyclic groups containing each other. Quaternion groups, dihedral groups
4. Generated subgroup, two descriptions, generating set, $d(G)$, examples
5. Symmetric group, product of disjoint cycles, product of transpositions. Special generating sets: $(1\ i)$ transpositions, $(i\ i+1)$ transpositions, 2-element generating set. Odd and even permutations, A_n
6. Cosets of a subgroup, index, Lagrange's theorem, order of $g \in G$ divides the order of G
7. Normal subgroup, conjugation, conjugacy class, conjugate subgroup. Normaliser, centraliser, centre
8. Conjugation in S_n , restriction of conjugacy classes to A_n . A_n is simple
9. Homomorphism, kernel, image. Mono, epi, iso. Factor group, natural homomorphism, homomorphism theorem. Automorphism, inner automorphism. Isomorphism theorems. Characteristic subgroup
10. Direct product, outer/inner, unique expression. Direct product with several factors. Fundamental theorem of finite Abelian groups.
11. Cauchy's theorem. Permutation groups and group actions on a set. Orbit, stabiliser, transitivity, regularity
12. Sylow's theorems, applications. Finite and infinite p -groups. Centre, normal subgroups and maximal subgroups of finite p -groups
13. Commutator, derived subgroup, description. Normal chain, solvable group. Higher commutator subgroups, derived series, derived length. Dependency between the derived lengths of a group and subgroups, factor groups of it. Composition chain, composition factors, Jordan-Hölder theorem. Solvability theorems of Burnside, Zassenhaus and Feit-Thompson
14. Linear groups (GL, SL, PGL, PSL), Dickson's theorem, order of these linear groups over finite fields
15. Rings, basic properties, examples. If $|R| > 1$ and for $a \neq 0$ and for every b the equation $ax = b$ is solvable in R then R is a skewfield
16. Subring, ideal (left/right ideal), generated ideal, principal ideal, examples. Boolean ring, set ring, group ring, group algebra
17. Homomorphism, factor ring, homomorphism theorem, isomorphism theorems
18. Integral domain, field of fractions. Divisibility and ideals. Unit, associates, primes irreducibles. UFD and its description using maximal condition on principal ideals. PID, ED. Greatest common divisor and its expression, examples
19. Characteristic of a field, prime field. Field extension, algebraic and transcendental elements, their characterisations. Abstract extension field using a root of an irreducible polynomial.