

1. Solve the following systems of linear equations!

$$\begin{array}{lll} \text{a)} & x+ & y+ & z = 10 \\ & x+ & 2y+ & 3z = 23 \\ & x+ & 4y+ & 9z = 59 \end{array} \quad \begin{array}{lll} \text{b)} & x- & y- & z = 1 \\ & x+ & y+ & 2z = 2 \\ & 5x+ & y+ & 4z = 3 \end{array} \quad \begin{array}{lll} \text{c)} & x- & y- & z = 1 \\ & x+ & y+ & 2z = 2 \\ & 5x+ & y+ & 4z = 8 \end{array}$$

2. Solve the following systems of linear equations in \mathbb{F}_5 !

$$\begin{array}{lll} \text{a)} & x+ & 2y+ & z = 4 \\ & x+ & 3y+ & 4z = 3 \\ & 2x- & y+ & 5z = 1 \\ & y+ & 7z = & 3 \end{array} \quad \begin{array}{lll} \text{b)} & -y+ & 2z+ & 3w = 1 \\ & 2x+ & 3y+ & 4z+ & 5w = 2 \\ & 2x+ & 2y+ & z- & 2w = 2 \end{array}$$

3. Which of the following are in (reduced) row echelon form? Compute the solutions of the corresponding equations (in parametric form)! A row echelon form is reduced, if all the pivot numbers are 1.

$$\begin{array}{lll} \text{a)} & \left(\begin{array}{cccc|c} 2 & 0 & 1 & 1 & 3 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{array} \right) & \text{b)} & \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array} \right) & \text{c)} & \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array} \right) \\ \text{d)} & \left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) & \text{e)} & \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

4. Solve the following systems of linear equations!

$$\begin{array}{ll} \text{a)} & \begin{array}{lll} x+ & y+ & z = 4 \\ -x+ & y- & z = 2 \\ 2x+ & y+ & 2z = 1 \\ 4x+ & 4y+ & 4z = 1 \end{array} & \text{b)} & \begin{array}{lll} 7x+ & 14y- & 21z = 7 \\ x+ & 2y- & 3z = 1 \\ 5x+ & 10y+ & 15z = 1 \\ 3x+ & 6y- & 9z = 3 \end{array} \end{array}$$

Can we leave some of the equations such that the system remains equivalent? Which can be left?

5. Does there exist a system of linear equations such that

- the number of equations is 5, the number of variables is 6 and there is a unique solution;
- the number of equations is 6, the number of variables is 5 and there is a unique solution;
- the number of equations is 5, the number of variables is 6 and there is no solution;
- the number of equations is 5, the number of variables is 5 and there are exactly 5 solutions

6. How many solutions does the following system of linear equations have depending on the values of a and b ? Solve the problem over \mathbb{R} , \mathbb{F}_2 and \mathbb{F}_3 !

$$\begin{array}{lll} x+ & y & = 1 \\ x+ & 2y- & az = b \\ x+ & 3y+ & az = 0 \end{array}$$