1. Recall the definition of orderings and well-ordered sets from the lecture!

(i) Show that an ordered set $(X, <)$ is well ordered if and only if it does not contain an infinite descending chain $(x_1 > x_2 > x_3 > \dots)$.

(ii) Prove that if S is well ordered, then all of its subset are well ordered!

(iii) Show that if $S_1, S_2, \dots S_n$ are well ordered subsets of an ordered set $(X, <)$, then so is $\bigcup_{k=1}^{n} S_k$.

(iv) Let $S_1, S_2, \dots$ be an infinite chain of well ordered subsets of an ordered set $(X, <)$ such that for all $i < j$, $x \in S_i$ and $y \in S_j$ we have $x < y$. Prove that $\bigcup_{k=1}^{\infty} S_k$ is also well ordered.

(v) Show that $\mathbb{Z}$ (with the standard ordering) is not well ordered and find an ordering $\prec$ such that $(\mathbb{Z}, \prec)$ is well ordered!

2. Compute the following divisions with remainder:

(i) $5 : 1$

(ii) $1 : 5$

(iii) $-333 : 5$

(iv) $36357478585 : 10$

(v) $2^{12} + 1 \ : 2$

(vi) $3^{12} - 1 \ : 3$

(vii) $10^{12} : 9$