

1. What is the algebraic form $(a + bi)$ of the following complex numbers?
a) $(3 - 4i)(7 + 8i)$, b) $(3 - 4i)/(2 - i)$, c) i^{2020} and d) $(1 + i)^9$.
2. Let $z = 1 + 3i$ and $w = 2 - i$. Compute
a) $z\bar{z}$, b) w/\bar{w} , c) $|z - w|$, d) $|2z - zw|$ and e) $|w/z\bar{w}^3|$.
3. What are the square roots of the complex number $1 - 2i$? (i. e. the numbers $w = x + yi$ such that $w^2 = 1 - 2i$)
4. Solve the equation $z^2 + 2iz - 1 + i = 0$ in \mathbb{C} !
5. Represent the solutions of the following equations on the plane!
a) $|z - 5 + i| = 2$ b) $|z - i| = |z + i|$ c) $|(z - 3 + 4i)/(z - i)| \geq 1$
d) $|z| = 3iz$ e) $|z| = iz$ f) $z + \bar{z} < 4$.
6. What are the fifth roots of $-\sqrt{3} + i$?
7. Give an explicit formula for $\binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \dots$ by comparing the algebraic and trigonometric form of $(1 + i)^n$!
8. Compute $(\cos x + i \sin x)^3$ in two different ways! With the help of this express $\cos(3x)$ as a function of $\cos x$!
9. We know that 2 and i are two vertices of a square. What can be the other two vertices?
10. Let z_1, z_2 and $z_3 \in \mathbb{C}$ and $\varepsilon_3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$. Prove that the following are equivalent:
(i) $z_1 z_2 z_3$ is an equilateral triangle with vertices in counterclockwise order and
(ii) $z_1 + z_2 \varepsilon_3 + z_3 \varepsilon_3^2 = 0$.
11. Let ε be a primitive n -th root of unity. What are the possible orders of
a) $-\varepsilon$ and b) ε^k ?
12. What is the sum and the product of primitive 5th and 8th roots of unity?