1. What is 2^{67} modulo 61?

2. Solve the congruences (in $\mathbb{Z}$)

(i) $12x \equiv 15 \pmod{21}$,

(ii) $12x \equiv 4 \pmod{6}$,

(iii) $12x \equiv 4 \pmod{2}$ and

(iv) $30x \equiv 4 \pmod{37}$.

3. Solve the following system of congruences:

$$\begin{aligned} x &\equiv 2 \pmod{3}; \\ x &\equiv 3 \pmod{8}; \\ x &\equiv -4 \pmod{11}. \end{aligned}$$

4. What is $5^{-1} \pmod{26}$?
Determine the invertibele elements modulo 26 and their inverses?

5. Compute the table of the operations of $\mathbb{Z}_3$ and $\mathbb{Z}_8$! What is the table of multiplication in $\mathbb{Z}_8^*$ (the invertible elements modudo 8)?

6. Determine the following values:

(i) $\varphi(23)$, $\varphi(21)$, $\varphi(63)$ and $\varphi(10!)$,

(ii) $120^{24} \pmod{23}$, $115^{21} \pmod{21}$, $68^{111} \pmod{63}$ and $111^{68} \pmod{63}$

(iii) the last two digits of $3^{3^{3^4}}$.

7. For which positive integers n is $\varphi(n) = 6$?

8. Euclides proved that there were ininitely many primes: If there are only finitely many, $p_1, p_2, \ldots, p_k$, then $N = p_1 p_2 \cdots p_k + 1 > 1$ cannot have a prime divisor, a contradiction.
Based on his idea, prove that there are infinitely many primes of the form $4k-1$ and of $3k-1$.
Modify the argument to prove that there are infinitely many primes of the form $4k+1$ and $3k+1$.

9. For a positive integer n let $f_1(n)$ denote the square of the sum of the decimal digits of n. From then on let $f_k(n) = f_1(f_{k-1}(n))$ for every $k \geq 2$. Determine $f_{2022}(2^{2021})$. (*based on a Kürschák competition question*)