**1.** Show that $641 | 2^{32}+1$ using that $641 = 5^4 + 2^4 = 5 \cdot 2^7 + 1$. (Euler)

**2.** Show that for positive odd integers $a, b$ we have $(2^a - 1, 2^b + 1) = 1$.

**3.** Show that for positive integers $a, b$ we have $(3^a - 1, 3^b - 1) = 3^{(a,b)} - 1$.

**4.** Show that if $a$ and $b$ are coprime (i.e. $(a, b) = 1$) and $ab$ is a square then $a$ and $b$ are both squares. Show that if $x$ is an even square then $x - 1$ is not a cube (except, of course, for $-1 = (-1)^3$).

**5.** Show that for $a, b$ integers we have $(a, b)[a, b] = ab$ and that $[a, b]$ divides every common multiple of $a$ and $b$.

**6.** Show that the following two congruences are equivalent for the integers $a, b, c, m$ (where $m > 0$):

$$ac \equiv bc \pmod{m} \quad \Leftrightarrow \quad a \equiv b \pmod{\frac{m}{(c, m)}}.$$

**7.** Give all integral and all positive solutions of the following linear equations!

(i) $288x + 204y = 30$ and

(ii) $288x + 204y = 300$.

**8.** Describe the solutions of the following linear congruences!

(i) $288x \equiv 30 \pmod{204}$,

(ii) $288x \equiv 300 \pmod{204}$ and

(iii) $204y \equiv 300 \pmod{288}$.

**9.** What is the decimal (base 10) representation of the number $\overline{120201}_3$?

**10.** Change the base from decimal!

(i) Convert 26 to base 16, 8, 4, 2, 5, 26!

(ii) Convert 2020 to base 2, 8 and 16!

**11.** What is the number $\overline{ab}_7 = \overline{ba}_{10}$?

**12.** Is there a limit on the length of a number that is in base 10 and equal to some rearrangement of its digits in base 7? (In the previous exercise we had length 2.)