1. Decide which of the four properties (reflexive, symmetric, transitive, antisymmetric) are true for the following relations.

1. the usual $\leq$ among the real numbers;
2. the usual $<$ among the real numbers;
3. the usual $=$ among the real numbers;
4. aRb if a and b have the same residue modulo 3 among the integers;
5. aRb if $a = b^2$ among the real numbers;
6. aRb if the reduced quotient $\frac{a}{b}$ has odd denominator among the integers;
7. the usual $|$ (divisible) among the integers;
8. aRb if a is the son of b among people;
9. aRb if a and b have a border crossing among countries;
10. aRb if we can travel from a to b on dry land among cities;
11. (in the last three the set in question is the set of all circles on the plane) c_1Rc_2 if c_1 and c_2 meet;
12. c_1Rc_2 if c_1 contains the centre of c_2;
13. c_1Rc_2 if c_1 contains c_2 or c_2 contains c_1.

2. Suppose that for the finite sets A, B there is a bijective function $f : A \to B$. Show that $|A| = |B|$.

3. Let A be a finite set of size a and B a finite set of size b. What is the cardinality of $A \times B$?

4. We are managing a hotel in Eastern Siberia with infinitely many rooms. The rooms are numbered $1, 2, \ldots$. Due to the high season, our hotel is fully booked, all rooms are occupied. Unexpectedly, an old man comes up to our doorstep begging for night shelter. Show that we can accomodate him without disposing of any of our current clientele.

Suddenly, there is a full bus arriving at our doorstep with infinitely many passengers (all numbered by their seat number $1, 2, \ldots$), how can we make room for them?

Finally, at midnight, a whole (infinite, of course) queue of full infinite buses appears at our doorstep and we feel we have to fit all the passengers in. Can we even achieve this?

5. Construct two functions $f, g : \mathbb{R} \to \mathbb{R}$ such that f is injective but not surjective and g is surjective but not injective.

6. Let R be a binary relation on X. We define a new relation R' by $aR'b$ if aRb or bRa holds. Show that R' is symmetric. Which properties out of the four above are inherited from R to R'?

7. Give a handful of "real life" examples for ternary relations. The more natural, the better.

8. Which (nonempty) subsets of $\mathbb{Z}$ are closed under addition and subtraction?

9. Consider a set S and a binary operation $*$, i.e., for each $a, b \in S$, $a*b \in S$. Assume $(a*b)*a = b$ for all $a, b \in S$. Prove that $a * (b * a) = b$ for all $a, b \in S$. (Putnam)