1. Solve the following systems of linear equations simultaneously.

$$\begin{cases} 2x- & y+ & z= & 1 \\ x+ & y+ & 2z= & 2 \\ x- & 2y- & z= & -1 \end{cases} \qquad \begin{cases} 2x- & y+ & z= & 1 \\ x+ & y+ & 2z= & 2 \\ x- & 2y- & z= & 0 \end{cases}$$

What does it mean from the point of view of rows/columns?

2. Let $\underline{v}_1, \underline{v}_2, \underline{v}_3 \in \mathbb{R}^4$ for which $\underline{v}_3 = \underline{v}_1 - 2\underline{v}_2$ and $\underline{v}_1$ is not a multiple of $\underline{v}_3$. What is the reduced row echelon form of $A = (\underline{v}_1, \underline{v}_2, \underline{v}_3) \in M_{4,3}(\mathbb{R})$?

3. Let

$$a = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, c = \begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, w = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \in \mathbb{R}^4.$$

Show that a, b, c are independent! Which of v and w is a linear combination of a, b and c? For those which are, compute the coefficients!

4. What is the rank of the matrix A and B, where $[A]_{jk} = j + k$ and $[B]_{jk} = j^2 + k^2$?

5. Let $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, C = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 3 & 5 \end{pmatrix}, D = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 5 \\ 0 & 0 & 7 \end{pmatrix}$.
Compute those, which are defined and determine their ranks:

$$A+B, A+C, A+D, B+A, B+D, AB, BA, AC, CA, CD, DC, A^2, B^2, C^2, D^2.$$

6. What are the row and column operations corresponding to the following matrices? (That is, when we multiply by them from left/right.)

$$\begin{pmatrix} 3 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

7. Do there exist matrices $C \in M_2(\mathbb{R})$ and $D \in M_3(\mathbb{R})$ such that $C \neq 0 = C^2$ and(or) $D^2 \neq 0 = D^3$?

8. Do the following equalities hold for all matrices $A, B \in M_n(F)$?
a) $(A+B)(A-B) = A^2 - B^2$ b) $(A+I)(A-I) = A^2 - I^2$
c) $(A+B)^2 = A^2 + 2AB + B^2$ d) $(AB)^T = A^T B^T$