**1.** As we easily check, 3 divides 1110, 2136, 5121 and 3987. Lo and behold, 3 divides

$$\begin{vmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 3 & 6 \\ 5 & 1 & 2 & 1 \\ 3 & 9 & 8 & 7 \end{vmatrix}.$$

Exlain why. Can we generalise to other numbers in place of 3?

**2.** Determine the following determinant and generalise!

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$$

**3.** Determine the determinant of $A = (a_{ij})$ if

(i) $a_{ij} = \min(i, j)$;

(ii) $a_ij = ij$;

(iii) $a_ij = i + j$

(iv) $a_ij = i^j$.

**4.** Given $A$, an $n \times n$ matrix, we obtain $B$ by excanging $a_{11}$ with $a_{12}$. Show that $\det(A) = \det(B)$ if and only if $a_{11} = a_{12}$ or $A_{11} = A_{12}$.

**5.** Let $a, b$ be complex numbers and $k$ a positive integer. Determine the determinant of the $2k \times 2k$ matrix whose main diagonal consists of $a$'s and the other diagonal consists of $b$'s, all other entries are 0.

**6.** Let $p_1 \cdots p_n$ be arbitrary and form the matix $A = (a_{ij})$, where

$$a_ij = \begin{cases} 1 - p_i^2, & \text{if } i = j; \\ -p_i p_j, & \text{if } i \neq j. \end{cases}$$

**7.** Show that $V(x_1, \ldots, x_n)$ is not a symmetric polynomial in the variables $x_1, \ldots, x_n$, however its square is. For which permutations $\sigma$ of $\{1, 2, \ldots, n\}$ is $V(x_1, \ldots, x_n) = V(x_{\sigma(1)}, \ldots, x_{\sigma(n)})$?

**8.** Show that if $a_1, \ldots, a_n$ are integers then $V(a_1, \ldots, a_n)$ is divisible by $V(1, \ldots, n)$.

**9.** Let $E_{ij}$ denote the matrix with 1 in the $i$-th row $j$-th volumn and 0 everywhere else. Determine $E_{ij}A$ and $AE_{ij}$ for a matrix $A$.

**10.** Which statements are true for matices $A, B$ where the products make sense?

(i) If $A$ has a full 0 row then $AB$ also has one.

(ii) If $A$ has a full 0 row then $BA$ also has one.

(iii) If all rows of $A$ form an arithmetic progression then so are the rows of $AB$.

(iv) If all rows of $A$ form an arithmetic progression then so are the rows of $BA$.

(v) It he sum of every entry of $A$ is 0 then so is for $AB$.

(vi) It he sum of every entry of $A$ is 0 then so is for $BA$.