1. Recall: Field extensions, degree, algebraic elements, splitting field.

2. Algebraically closed field, algebraic closure. Existence and uniqueness.

3. Galois group, Galois connection, Galois extension. Permutation of the roots of a polynomial.

4. Normal extension and its equivalent characterisations.

5. Separability: of a polynomial, an element and an extension.

6. Lemma when an orbit of an element is finite under the action of a Galois group. Characterisation of Galois extensions.

7. Primitive Element Theorem and its corollary on the order of a Galois group of finite extensions. Finite subgroups of field automorphism groups.

8. Fundamemntal Theorem of Galois Theory. Examples.

9. Cyclotomic polynomials. Gauss' Theorem of irreducibilty.

10. Inverse Galois problem for Abelian groups. Special case of Dirichlet's Theorem on primes in arithmetic progressions.

11. Finite Fields. Their sizes, containment, separability and splitting polynomials. Galois groups. Irreducible polynomials of arbitrary degree.

12. Solvability by radicals. "Most" Galois groups are S_n or A_n, example for S_p. Chebotarev's Density Theorem.

13. Modules. Sub- and factormodules, direct sums and free modules. Homomorphism theorems. Submodules of free modules over PID's, their factor modules and the Smith Normal Form. Invariant factors and elementary divisors. Fundamental Theorem of Finitely Generated Modules over Principal Ideal Domains. Applications for $\mathbb{Z}$ and for $F[x]$.

14. Integral elements of rings over subrings. Module theoretic characterisation. Ring of integral elements. Conceptual parallels with algebraic extensions. Integral closure in an overring, integrally closed rings.

15. Ring of algebraic integers, Ω. Number fields, $\mathcal{O}(\mathbb{Q}(\alpha))$. Integrality and the minimal polynomial. Norm and trace. Units, associates, primes and irreducible elements. Infinitely many irreducibles.

16. Discriminant of a basis, integral basis, existence, discriminant of a number field. Quadratic fields, their characterisation with square free integers. Integral bases and discriminants.

17. Ideals in $\mathcal{O}(\mathbb{Q}(\alpha))$. Finite generation, ideal bases. Maximal and prime ideals. Division and containment. Gcd and lcm of ideals. Dedekind rings.

18. Euclid's Lemma, fractional (prime) ideal. $PP^{-1} = \mathcal{O}$. Unique factorisation of ideals into product of prime ideals. Rational primes in prime ideals, infinitely many prime ideals.

19. Congruences, Chinese Remainder Theorem. Exact division requirements. Every ideal is 2-generated, one of them may be a rational integer.

20. Norm of an ideal, multiplicativity. Norm of prime ideals, degree of a prime ideal. $N((\alpha)) = |N(\alpha)|$. Ideal products in normal number fields. $I = (\alpha, \beta)$, $N(I) = (N(\alpha), N(\beta))$. Factorisation of (p) in quadratic number fields.

21. Equivalence of ideals. Ideal classes. Ideal class group. Bound for the minimal norm of an ideal in a class. Examples.

22. Obtaining larger structures from smaller ones: $\mathbb{N} \to \mathbb{Z} \to \mathbb{Q} \to \mathbb{R} \to \mathbb{C}$. Cauchy sequences and null sequances. Completion with respect to $|\cdot|$. Archimedean and non-Archimedean valuations of $\mathbb{Q}$, $|\cdot|_p$. Field of p-adic numbers: completion with respect to $|\cdot|_p$. $\mathbb{Z}_p$, the p-adic integers and their representation as sequences $(a \bmod p, a \bmod p^2, a \bmod p^3, \ldots)$.